P] IASE NOISE IN PI IOTONIC PI IASED-ARRAY ANTI:NNA SYSTEMS"

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Abstract

The total noise of a phased-array antenna system employing a photonic feed net work is anal yzed using a model for the individual component noise including both additive and multiplicative equivalent noise generators. Additive noise is present independent of signal amplitude, whereas mul tiplicat ive noise is onl y present in proportion to the signal amplit ude. Thermall y-generated amplifier noise and laser relative-intensity-noise (RI N) are examples of additive noise; gain or phase instabilities are examples of multiplicative noise. It is shown that uncorrelated multiplicative noise power of the individual feeds is reduced by a factor of N in the output of an N-element linear array. However, the uncorrelated additive noise of the individual feed paths is not mitigated, and therefore will determine the minimum noise floor of a large phased-array.

In phased-array antennas, the beam quality depends critically on the phase control of the signals at the individual antenna elements. The ability to feed and adjust the phase of the microwave signals to the individual radiating elements using optical fiber and photonic components offers obvious advantages in size, weight, mechanical flexibility, and cross-talk, compared to metallic waveguides and phase-shifters. Various phased-array antenna system architectures with photonic feed networks have been proposed, however, the issues of phase stability and signal purity are not typically addressed in these proposals. Therefore, determination of the acceptable phase noise contribution of the individual active feed components has been problematic. In this paper, the general factors contributing to the phase stability of an array feed network are outlined, with particular attention paid to the type of noise encountered in photonic feed elements, It is shown that the analysis of array phase stability must consider both additive! and multiplicative noise generation processes, and that additive noise from active components will limit the phase stability of a large phased-array.

The general architecture of a phased-array radar system comprised of M elements is depicted schematically in Figure 1. In this analysis, the phase noise contribution due to the array feed and antenna elements only is calculated. The effects of the. source phase noise will be cm-m-non to all elements, and may be treated in the usual manner for a single antenna element.

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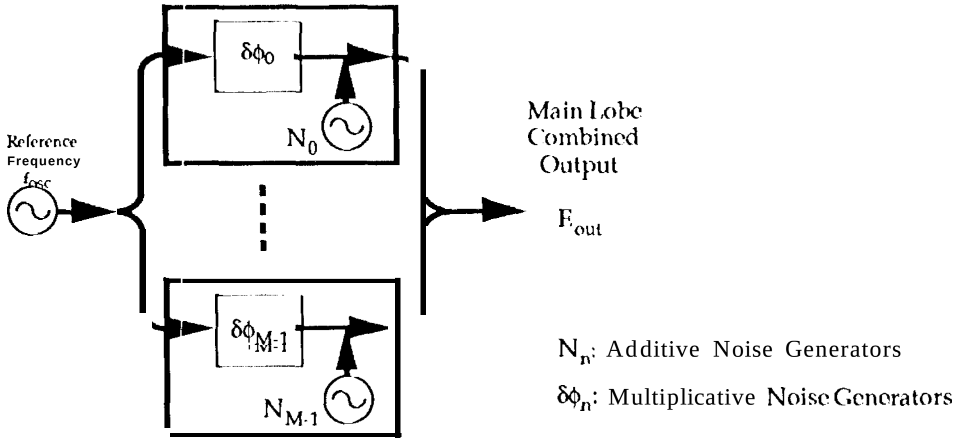

Figure 1. Noise Sources in M-1 Element Phased-Array Feed System

The photonic feed elements contain active components such as laser diodes, photodiodes, and amplifiers. The phase noise contribution of a microwave fiber optic feed system is therefore comprised of an additive noise term and a multiplicative noise term. Laser relative intensity noise (1<1 N), shot noise, and thermal (Johnson) noise are additive noise sources which are present at all times independent of signal level. Low frequency gain or path-length instabilities that modulate the microwave signal amplitude and phase are multiplicative **noise** sources that are only observed when a signal is present. As shown in Figure 2, additive noise usually determines the noise floor at higher offsets from the carrier frequency, but multiplicative! noise often has a $1/f^{\alpha}$ power law characteristic, $1<\alpha<2$, so is typically dominant close to the microwave carrier frequency [1 1.

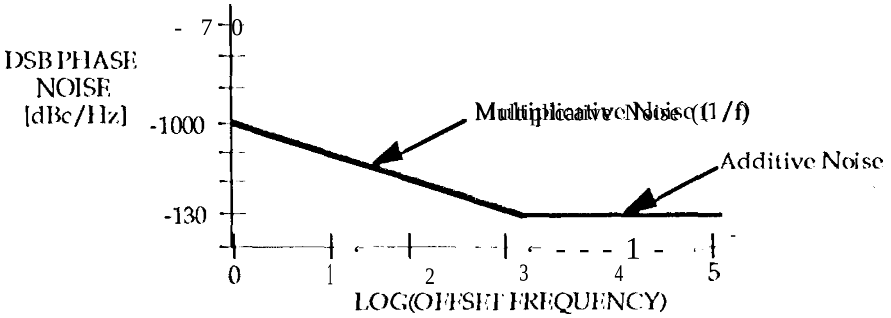

Figure 2. Typical Phase Noise of Photonic Feed System at 10 GHz

Additive noise sources due to thermal and shot processes or laser RIN are independent random processes and therefore can be assumed uncorrelated between the feed elements. Multiplicative noise may or may not be uncorrelated between elements, depending cm its origin. For example, thermal expansion or vibration of all the optical fibers in the feed network may produce a phase modulation that is common to all elements, whereas, laser or amplifier-induced 1/f gain fluctuations will be uncorrelated between elements. Noise that is common to all the elements may be referred to the source oscillator and treated as if the array were a single element. in the analysis that follows, all of the noise sources in the individual feed elements are assumed to arise from independent random processes, so are uncorrelated.

It is noted that the multiplicative noise is not detectable by a standard noise figure measurement in the microwave signal frequency band, In fact, it **is** difficult in practice to predict the amount of multiplicative noise in an amplifier or laser diode, because the noise level may itself be a function of the modulation signal frequency or amplitude. Therefore, the amount of multiplicative phase noise is usually determined empirically.

The effect of the individual feed element noises on the total array noise may be computed by including the additive noise $N_n(t)$ and multiplicative noise $\delta\phi_n(t)$ terms in the standard calculation [2] of the array output field. Considering a linear array at a steering angle ϕ, the field distribution in the far field of the array as a function of observation angle θ is expressed by

$$E_{out}(\theta,\phi,t) = \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1}\left(\frac{E_{in}(t)e^{jn(\pi\cos\theta-\phi)}e^{j\delta\phi_n(t)}}{\sqrt{M}} + N_n(t)\right). \qquad (1)$$

The peak of the antenna pattern main lobe occurs where $\pi\cos\theta = \phi$. For small-angle phase noise $\delta\phi(t) << 1$ radian, the time variation of the output field at the peak of the main lobe can be written

$$E_{out}(t) = \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1}\left(\frac{E_{in}(t)(1 + j\delta\phi_n(t))}{\sqrt{M}} + N_n(t)\right). \qquad (2)$$

To illustrate the effect of the parallel noise sources on the output, first consider the output of an "array" comprised of only one antenna element with unity gain. In the above equation, this corresponds to the case where M=1. The output field is then given by

$$E_{out}(t) = E_{in}(t) + j\delta\phi(t)E_{in}(t) + \mathrm{N(f)}. \qquad (3)$$

The signal and noise power are proportional to the **time-average** of the squared-magnitude of the output voltage:

$$P_{out} \propto \left(\begin{array}{l} \langle E_{in}E_{in}^*\rangle + \langle E_{in}(j\delta\phi)^* E_{in}^*\rangle + \langle E_{in}N^*\rangle \\ + \langle j\delta\phi E_{in}E_{in}^*\rangle + \langle j\delta\phi E_{in}(j\delta\phi)^* E_{in}^*\rangle + \langle j\delta\phi E_{in}N^*\rangle \\ + \langle NE_{in}^*\rangle + \langle N(j\delta\phi)^* E_{in}^*\rangle + \langle NN^*\rangle \end{array}\right) \qquad (4)$$

where the angle-brackets ( ) denote the time-average of the enclosed quantity, and the explicit time-dependence of the functions has been dropped for clarity. For independent zero-mean noise processes, the time-average of products of the constant and random terms are zero, because wc have assumed zero-mean random noise processes. But the time-average of the square of any noise term is non-zero. Thus, the ratio of signal to noise power for one antenna is

$$SNR\big|_{1-element} = \frac{\left\langle |E_{in}(t)|^2 \right\rangle}{\left\langle |j\delta\phi(t)E_{in}(t)|^2 \right\rangle + \left\langle |N(t)|^2 \right\rangle} \tag{5}$$

Similarly, for the case of M antennas with independent equal-amplitude multiplicative and additive noise sources, the output field is given by

$$\begin{aligned} E_{out}(t) &= \frac{1}{\sqrt{M}}\left(\frac{E_{in}(t)}{\sqrt{M}}\left(1 + j\delta\phi_0(t)\right) + N_0(t) + \frac{E_{in}(t)}{\sqrt{M}}\left(1 + j\delta\phi_1(t)\right) + N_1(t) + \cdots\right) \\ &= E_{in}(t) + \frac{E_{in}(t)}{M}\left(j\delta\phi_0(t) + j\delta\phi_1(t) + \cdots\right) + \frac{N_0(t)}{\sqrt{M}} + \frac{N_1(t)}{\sqrt{M}} + \cdots \end{aligned} \tag{6}$$

The individual powers of equal-amplitude, independent (therefore uncorrelated) noise sources may be added linearly. Thus, the equivalent noise voltage due to the sum of M uncorrelated noise sources can be represented as $\sqrt{M}$ times the amplitude of a single noise source. Since all cross-terms between uncorrelated noise sources will average to zero, we can write the output field in terms of a single multiplicative noise source $\delta\phi(t)$ and a single additive noise source $N(t)$

$$E_{out}(t) = E_{in}(t) + \frac{E_{in}(t)j\delta\phi(t)}{\sqrt{M}} + N(t). \tag{7}$$

Now, the ratio of signal to noise power in the combined output of an M-element array is

$$SNR\big|_{M-elements} = \frac{\left\langle |E_{in}(t)|^2 \right\rangle}{\frac{\left\langle |j\delta\phi(t)E_{in}(t)|^2 \right\rangle}{M} + \left\langle |N(t)|^2 \right\rangle}. \tag{8}$$

So, it is seen that the multiplicative noise is mitigated by a factor of M in the combined output, whereas the signal power and additive noise power are unchanged from the single-element case.

in summary, it was shown that as the number of array elements M is increased, the effect of uncorrelated multiplicative phase noise of the feed elements on the total array stability is diminished. However, the uncorrelated additive noise of the feed elements is not diminished, so that the signal-to-noise ratio becomes independent of array size for large M. It is therefore important to quantify both the additive and multiplicative noise of the feed elements to correctly predict the total array phase stability.

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